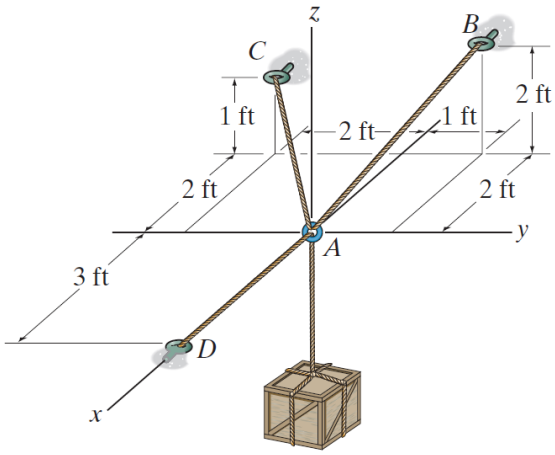
Problem 3-66

Determine the tension developed in cables AB, AC, and AD required for equilibrium of the 300-lb crate.



Prob. 3–66

Solution

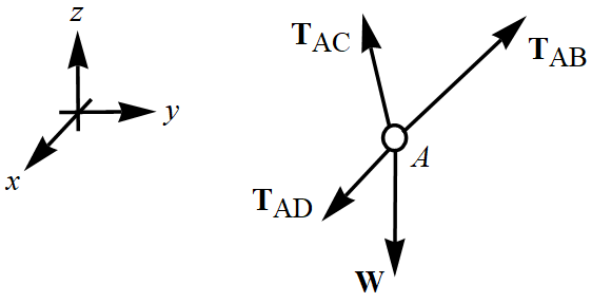
Write position vectors to points A, B, and C.

$$\mathbf{r}_A = \langle 0, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle -2, 1, 2 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -2, -2, 1 \rangle \text{ ft}$$

Draw a free-body diagram for point A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{W} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\langle 1, 0, 0 \rangle + W\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{\text{AC}}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{\text{AD}}\langle 1, 0, 0 \rangle + W\langle 0, 0, -1 \rangle = \mathbf{0}$$

Write out the unit vectors and simplify the left side.

$$T_{\text{AB}}\frac{\langle -2-0, 1-0, 2-0\rangle}{\sqrt{(-2-0)^2+(1-0)^2+(2-0)^2}} + T_{\text{AC}}\frac{\langle -2-0, -2-0, 1-0\rangle}{\sqrt{(-2-0)^2+(-2-0)^2+(1-0)^2}} + T_{\text{AD}}\langle 1,0,0\rangle + W\langle 0,0,-1\rangle = \mathbf{0}$$

$$T_{\text{AB}}\left\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle + T_{\text{AC}}\left\langle -\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right\rangle + T_{\text{AD}}\langle 1,0,0\rangle + W\langle 0,0,-1\rangle = \mathbf{0}$$

$$\left\langle -\frac{2}{3}T_{\text{AB}} - \frac{2}{3}T_{\text{AC}} + T_{\text{AD}},\ \frac{1}{3}T_{\text{AB}} - \frac{2}{3}T_{\text{AC}},\ \frac{2}{3}T_{\text{AB}} + \frac{1}{3}T_{\text{AC}} - W\right\rangle = \langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} -\frac{2}{3}T_{\text{AB}} - \frac{2}{3}T_{\text{AC}} + T_{\text{AD}} &= 0 \\ \frac{1}{3}T_{\text{AB}} - \frac{2}{3}T_{\text{AC}} &= 0 \\ \frac{2}{3}T_{\text{AB}} + \frac{1}{3}T_{\text{AC}} - W &= 0 \end{aligned}\right\}$$

Solve it for T_{AB}, T_{AC}, and T_{AD} and then plug in $W = 300$ lb.

$$T_{\text{AB}} = \frac{6}{5}W = 360.\ \text{lb}$$

$$T_{\text{AC}} = \frac{3}{5}W = 180.\ \text{lb}$$

$$T_{\text{AD}} = \frac{6}{5}W = 360.\ \text{lb}$$